IF and Epistemic Action Logic

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Abstract

This paper is a tentative discussion about the alleged opposition between van Benthem's account of logic games (Epistemic Action Logic, EAL) and Hintikka's IF first-order logic. Both logics provide a specific viewpoint on the relationship between logic and games, especially games of imperfect information. Arguing for a cooperative vs. competitive strategy, we will combine EAL and IF logic and obtain natural formulations of the existence of a uniform winning strategy for the verifier in an evaluation game. One of the upshots of the combination IF-EAL is a well known result usually put forward about IF-languages, namely that a sentence is equivalent to its truth-conditions, formulated in a new frame.

1 Introduction

In this paper I will explore several features of the interrelation between logic and games, as they are conceived of according to two radically different frames: Hintikka's IF first-order logic (IF-FOL) and van Benthem's Epistemic Action Logic (EAL). EAL appears to be a young and serious challenger for IF-FOL since it provides a very sharp account of imperfect-information games. One of the goals of this paper is to show that both approaches should be taken for complementary views likely to mutually enrich one another rather than as irreducibly rival conceptions.

After a short presentation of both logics (Sections 2 and 3), we will deal with the question whether IF-FOL is reducible to EAL in some sense, or not. From an IF-FOL sentence φ, a model (or 'game board') $\mathbf{M}$ and an assignment s, one can build the evaluation game of φ in the given model relative to s: $game(\varphi, \mathbf{M}, s)$. Standard EAL then enables to describe the corresponding game tree. Does EAL

*I am greatly indebted to Johan van Benthem's critical comments and constructive ideas, without which this paper could not have been written down. I also wish to thank Tero Tulenheimo for his very useful comments. All errors remain mine, though.

enable a description of every relevant property of the game? Unfortunately, it is in general not the case.

Fact 1 *Standard EAL cannot express that* there is a uniform winning strategy for the verifier in $game(\varphi, \mathbf{M}, s)$.

An example is provided where standard EAL can express that there is a winning strategy at the beginning of the game, but cannot define that there is no *uniform* winning strategy. In the game-theoretical frame, this is an important fact: the existence of a uniform winning strategy for the verifier in $game(\varphi, \mathbf{M}, s)$ indeed constitutes the truth-condition for φ.
Using Hintikka's idea that IF-FOL has a truth definition inside IF, but not inside standard FOL, I will propose to translate it to EAL and to consider an IF extension EAL (Section 4). In this new logic one can assert that there is a *u*niform *w*inning *s*trategy for the verifier in $game(\varphi,\ \mathbf{M},\ s)$. Let's denote by $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$ this IF-EAL formula. What is expected is that $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$ is true at the root of $game(\varphi,\ \mathbf{M},\ s)$ iff φ is true at $\mathbf{M},\ s$:

$$game(\varphi, \mathbf{M}, s), \mathbf{root} \Vdash \mathbf{uws}(game(\varphi, \mathbf{M}, s)) \Leftrightarrow \mathbf{M}, s \vDash \varphi \tag{1}$$

Of course, there is no standard way to evaluate IF-EAL formulas but there is a natural resort to games for IF languages in general. I will thus propose to use new evaluation games. Now there is an interesting fact about the evaluation game of $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$:

Fact 2 *At the root of $G = game(\varphi, \mathbf{M}, s)$, the evaluation game of $\mathbf{uws}(G)$ is isomorphic to the original game G:*

$$game(\mathbf{uws}(G), G, \mathbf{root}) \cong G. \tag{2}$$

Were both games only bisimilar, the conclusion to be drawn would have been that they (their roots) would share the same *standard* EAL formulas; as they are isomorphic, they also share IF-EAL formulas. As a consequence, **uws**(G) is true at the roots of *both* games, not only of the original one G; **uws**(G) is thus true at the root of its own evaluation game:

Fact 3 *$G = game(\varphi, \mathbf{M}, s)$ is enough – i.e. in order to see whether the verifier has a uniform winning strategy in $game(\mathbf{uws}(G), G, \mathbf{root})$, no more 'meta game' is needed.*

Hence IF-EAL can put an end to some fearsome infinite regression between IF and EAL. This is just the translation of Hintikka's idea that truth for an IF language can be defined within the very same language.
Besides this result, a few issues connected to IF-EAL will be discussed in Section 5. Thanks to IF-EAL we will obtain a kind of 'equivalence' between an IF-FOL sentence φ and a corresponding IF-FOEL (IF first-order *epistemic* logic)

formula, **egof**(φ, **M**, s), stating that in the evaluation game *game*(φ, **M**, s) the verifier knows which strategy is a winning strategy for herself. If one takes games and players at face value, this equivalence appears to be a very natural one since it corresponds to the truth-definition for IF formulas, viz. the existence of a uniform winning strategy for the verifier, formulated in the frame of EAL.

2 IF First-Order Logic in a Nutshell

IF First-Order Logic (IF-FOL) was created by Hintikka and developed by Hintikka and Sandu in the 1990s as an extension of standard first-order logic (FOL). It is a quite natural extension when connected to Game-Theoretical Semantics (GTS). According to GTS, each FOL-formula φ is interpreted relatively to some model **M** through a specific game, *game*(φ, **M**), played between two abstract players, the initial verifier and the initial falsifier, s.t. the first player (resp. the second one) has a uniform winning strategy iff the formula is true (resp. false) in **M**. (A more fine-grained definition would add an assignment s and consider *game*(φ, **M**, s), but it is not essential here.) Such evaluation games are played according to the following rules:

- **(R.At)**. If A is a true atomic sentence (or identity), the verifier wins *game*(A, **M**) and the falsifier loses it. If A is a false atomic sentence (or identity), vice versa.
- **(R.$\vee$)**. In *game*($\varphi_1 \vee \varphi_2$, **M**) the verifier picks out an index $i \in \{1, 2\}$. The rest of the game is as in *game*(φ_i, **M**).
- **(R.$\wedge$)**. *game*($\varphi_1 \wedge \varphi_2$, **M**) is likewise, except that the choice is made by the falsifier.
- **(R.$\exists$)**. *game*(($\exists x$) $\varphi[x]$, **M**) begins with the choice by the verifier of a member of do(**M**) and of a name b; the rest of the game is as in *game*($\varphi[b]$, **M**).
- **(R.$\forall$)**. *game*(($\forall x$) $\varphi[x]$, **M**) is likewise, except that the falsifier makes the choice.
- **(R.$\sim$)**. *game*($\sim\varphi$, **M**) is like *game*(φ, **M**), except that the roles of the two players (as defined by these rules) are interchanged.

Games corresponding to standard FOL formulas are of course determined, so that the principle of excluded middle holds. Moreover, these are perfect-information games: both players know or remember what all the previous moves of the play are. The "natural extension" consists in considering *imperfect*-information games, i.e. games where the players lack some information about the actual play. Hintikka suggested considering the case where the initial verifier has to make (some of) her moves (i.e. according to (R.$\vee$) or (R.$\exists$)) while ignoring some prior moves of her opponent. Such informationally independent moves in the semantic interpretation are expressed by the slash-notation at the level of the language. For instance, while playing the game associated with $\forall x\ (\exists y/\forall x)\ \varphi[x,\ y]$, the initial verifier will have to choose a value for y independently from that (chosen by the falsifier) of x. Similarly, in the game correlated to $\forall x\ (\varphi_1\ (\vee/\forall x)\ \varphi_2)$, the verifier will choose a disjunct φ_i not knowing the value of x.

The introduction of the slash-notation at the syntactic level leads to a new logic, IF-FOL, which enables to tackle new patterns of mutual (in)dependence between quantifiers. A paradigmatic example is provided by the Henkin or branching quantifiers, such as:

$$\begin{smallmatrix}\forall x \exists y\\ \forall z \exists u\end{smallmatrix}\varphi[x, y, z, u] \tag{3}$$

Into IF-FOL this formula is rendered e.g. by $\forall x\exists y\forall z(\exists u/\forall x)\,\varphi[x, y, z, u]$, whereas it is not expressible in standard FOL. The new patterns can be made visible in the Skolem normal forms of first-order formulas, where existential quantifiers are replaced by function symbols whose variables are taken among the preceding universally quantified ones: an existential quantifier independent from some universal quantifiers will thus be replaced by a function without the corresponding variable. For example, whereas the Skolem normal form of $\forall x\exists y\forall z\exists u\,\varphi[x, y, z, u]$, (where φ is quantifier-free) will be $\forall x\ \forall z\ \varphi[x,\ \mathbf{f}(x),\ z,\ \mathbf{g}(x,\ z)]$, that of the IF formula $\forall x\ \exists y\ \forall z\ (\exists u/\forall x)\ \varphi[x,\ y,\ z,\ u]$ will be $\forall x\ \forall z\ \varphi[x,\ \mathbf{f}(x),\ z,\ \mathbf{h}(z)]$, i.e. the function replacing the independent quantifier is made independent from x.

Skolem functions and their analogues for disjunctions play a special role in IF-FOL, since they are natural candidates to encode the initial verifier's winning strategies. Now the GTS truth-conditions of IF or standard first-order formulas are straightforwardly expressible using Skolem normal forms by prefixing them with second-order existential quantifiers of the Skolem (or strategy) functions. For instance, the IF formula $\forall x\ \exists y\ \forall z\ (\exists u/\forall x)\ \varphi[x,\ y,\ z,\ u]$ is GTS-true in some model $\mathbf{M}$ iff there is a winning strategy for the initial verifier in the correlated game, which is expressed by the second-order and in fact Σ^1_1 sentence: $\exists \mathbf{f}\ \exists \mathbf{h}\ \forall x\ \forall z\ \varphi[x,\ \mathbf{f}(x),\ z,\ \mathbf{h}(z)]$.

Let's add a few comments. Besides what can be called the *model-denotation* of a FOL formula φ, (i.e. its standard model-theoretic semantic value: the set of models where φ is true), Hintikka's semantics thus provides a more fine-grained denotation, the *game-denotation* of φ which is the set of games associated with φ where the initial verifier has a uniform winning strategy. The latter notion is more fine-grained than the former one because two logically equivalent formulas will lead to two different classes of games. Furthermore, both kinds of denotations can be restrictively defined relative to a given model $\mathbf{M}$: the model-denotation is thus a set of denotations, whereas the game-denotation is a restricted set of games.

To put it in a reversed perspective: given a model $\mathbf{M}$, true FOL formulas *describe* it in the usual way but can hardly be said to 'describe' their evaluation games. However, the second-order formula $uws(game(\varphi,\ \mathbf{M},\ s))$ stating that there is a *u*niform *w*inning *s*trategy for the verifier in $game(\varphi,\ \mathbf{M},\ s)$ can be said to describe this game. The distinction between the two meanings of 'denotation' is very clear for standard FOL formulas. But in IF-FOL, the situation appears to be different. Indeed, IF-FOL is an extension of standard FOL which exactly coincides with the Σ^1_1 fragment of second-order logic. As a consequence, game-theoretical truth-conditions $uws(game(\varphi,\ \mathbf{M},\ s))$ of first-order sentences φ can be translated into IF-FOL, and in particular *IF formulas express their own truth-conditions*. It means that if φ is IF, it is identical with $uws(game(\varphi,\ \mathbf{M},\ s))$.

An IF-FOL formula φ can thus be considered for itself – with two denotations – or as a (second-order) assertion about its game-denotation. It does not mean that model- and game-denotations of IF formulas coincide, for model-denotations are classes of usual models whereas game-denotations are classes of games. However, relatively to a model $\mathbf{M}$ an IF formula φ is simultaneously an assertion about $\mathbf{M}$ (via its model-denotation) and an assertion about the class of games $game(\varphi, \mathbf{M}, s)$. Of course, this combination is not the case for standard FOL formulas. (I will go back to this reflexive aspect of IF-FOL in Section 5 below.)

3 Epistemic Action Logic in a Nutshell

Van Benthem's EAL is a competing frame to deal with imperfect-information games. More precisely: Whereas standard (i.e. game-theoretical) semantic interpretation associates imperfect-information games to IF-formulas, EAL is a dynamic epistemic language specially designed to describe the properties of those games. In fact, the starting point is different and the whole perspective is reversed: IF-FOL uses (evaluation) games as good tools for semantic interpretation whilst EAL considers games for themselves and aims to provide interesting insights on their properties.

Syntax Like Propositional Dynamic Logic, EAL is a dual language made of 'formulas' and 'actions' with mutual combination. The vocabulary of EAL will partly depend on the model on which the games described by the language are played. Hence relatively to some model $\mathbf{M}$ of domain do($\mathbf{M}$), formulas and actions can have the following syntactic forms:

- Atoms: $At = \{\mathrm{win}_{\mathbf{V}},\ \mathrm{turn}_{\mathbf{V}},\ \mathrm{win}_{\mathbf{F}},\ \mathrm{turn}_{\mathbf{F}}\}$
(the verifier $\mathbf{V}$ is winning, it is the verifier's turn, and the same for the falsifier $\mathbf{F}$)
- Basic actions: $B = \{x := a,\ x := b,\ \ldots,\ y := a,\ y := b,\ \ldots,\ \mathrm{L},\ \mathrm{R}\}$
(object picking – where $a, b\ldots$ are non-logical individual constants designating elements of do($\mathbf{M}$) –, going left, going right)
- Actions: $A ::= B \mid A{\cup}A \mid A\ ;\ A$
(basic actions, choice, composition[1])

[1]Kleene iteration 'π^*', and tests for formulas '$(\varphi)?$' are other kinds of actions. (Adding the 'demonic' duality-operation 'π^d' would yield a kind of Game Logic, see Pauly and Parikh

- Wffs: $F ::= At \mid \bot \mid \neg F \mid F \vee F \mid \langle A \rangle\ F \mid \mathrm{K}_{\mathbf{i}}\ F$
  (atoms, contradiction, negation, disjunction, action modality, epistemic modality)

Let's add a few comments:

- $\pi_1 \cup \pi_2$ is the (complex and) non-deterministic action consisting of the execution of π_1 or of π_2.
- π_1 ; π_2 is the action consisting of the execution of π_1, then of π_2.
- $\mathrm{K}_{\mathbf{i}}\ \varphi$ should be read as "**i** knows that φ", where $\mathbf{i} \in \{\mathbf{V}, \mathbf{F}\}$.
- $\langle \pi \rangle\ \varphi$ should be read as "some execution of π from the current node leads to a node where φ is true."
- We can define its dual, $[\pi]\varphi := \neg\langle\pi\rangle\neg\varphi$, and read it as "every execution of π from the current node leads to a node where φ is true."
- There are as many object-picking basic actions in B as elements in the domain do($\mathbf{M}$); consequently, when the model is infinite, so is the set of (basic) modalities.

Models Games in extensive form provide (regular) Kripke models for EAL:

$$\mathbf{G} = (\mathrm{W}, \{\mathrm{R}_\pi\}_{\pi \in A}, \{\sim_{\mathbf{V}}, \sim_{\mathbf{F}}\}, \mathrm{V})$$

where: W is a set of states (nodes); R_π is the binary accessibility relation encoding the transition for action of type π (with $\mathrm{R}_{\pi_1 \cup \pi_2} = \mathrm{R}_{\pi_1} \cup \mathrm{R}_{\pi_2}$ and $\mathrm{R}_{\pi_1;\pi_2} = \mathrm{R}_{\pi_1} \circ \mathrm{R}_{\pi_2}$); $\sim_{\mathbf{V}}$ and $\sim_{\mathbf{F}}$ are equivalence 'uncertainty' relations encoding the information sets for each player; V is a valuation function for the atoms.

Semantics Truth of formulas and successful executions of actions must be defined simultaneously. The truth of an EAL-formula φ at a node s of a game $\mathbf{G}$ is denoted by: $\mathbf{G}, s \Vdash \varphi$. The fact that a successful execution of the action π in a game $\mathbf{G}$ corresponds to a transition from s to t is denoted by: $\mathbf{G}, s, t \Vdash \pi$.

- Formulas:

$\mathbf{G}, s \nVdash \bot$		
$\mathbf{G}, s \Vdash p$	iff	$s \in \mathrm{V}(p)$ for $p \in At$
$\mathbf{G}, s \Vdash \neg\varphi$	iff	$\mathbf{G}, s \nVdash \varphi$
$\mathbf{G}, s \Vdash \varphi_1 \vee \varphi_2$	iff	$\mathbf{G}, s \Vdash \varphi_1$ or $\mathbf{G}, s \Vdash \varphi_2$
$\mathbf{G}, s \Vdash \langle\pi\rangle\varphi$	iff	there exists a node t s.t. $\mathbf{G}, s, t \Vdash \pi$ and $\mathbf{G}, t \Vdash \varphi$
$\mathbf{G}, s \Vdash K_i\varphi$	iff	$\mathbf{G}, t \Vdash \varphi$ for all node t s.t. $s \sim_i t$, where $i \in \{\mathbf{V}, \mathbf{F}\}$.

2003). Here I consider a simplified version of EAL without these operations, which is sufficient for the purpose of this paper.

- Actions:

$$\begin{array}{lll} \mathbf{G}, s, t \Vdash \pi & \text{iff} & (s,t) \in \mathrm{R}_\pi \\ \mathbf{G}, s, t \Vdash \pi_1 \cup \pi_2 & \text{iff} & \mathbf{G}, s, t \Vdash \pi_1 \text{ or } \mathbf{G}, s, t \Vdash \pi_2 \\ \mathbf{G}, s, t \Vdash \pi_1 ; \pi_2 & \text{iff} & \text{there exists a node } u \text{ s.t. } \mathbf{G}, s, u \Vdash \pi_1 \text{ and } \mathbf{G}, u, t \Vdash \pi_2 \end{array}$$

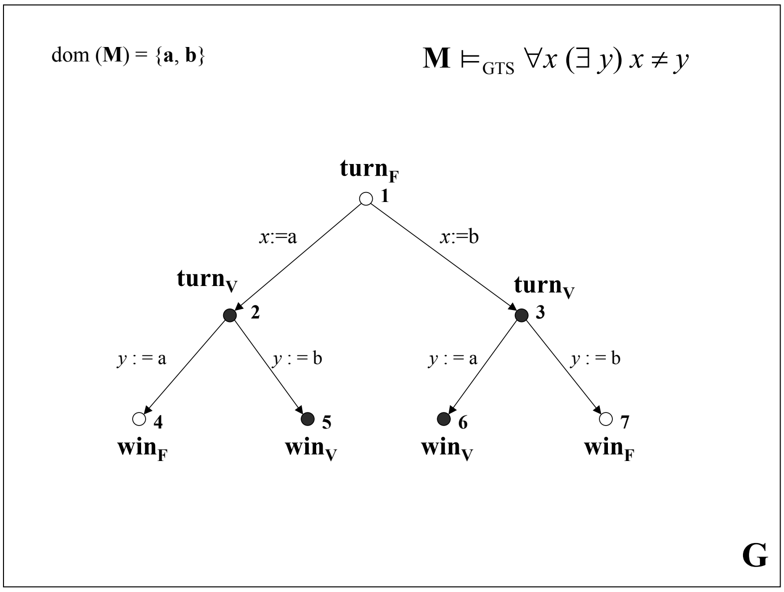
The following equivalences are obtained in a straightforward manner:

$$\begin{array}{lcl} \langle \pi_1 \cup \pi_2 \rangle \varphi & \Leftrightarrow & \langle \pi_1 \rangle \varphi \vee \langle \pi_2 \rangle \varphi \\ [\pi_1 \cup \pi_2] \varphi & \Leftrightarrow & [\pi_1] \varphi \wedge [\pi_2] \varphi \end{array}$$

Example 1. Let's consider the (GTS) evaluation game $\mathbf{G}$ of the standard first-order formula

$$\forall x \exists y \, (x \neq y) \tag{4}$$

on a two-element model, in extensive form:



In this example, one can easily check the following assertion:

$$\mathbf{G}, \mathbf{1} \Vdash [x := a \cup x := b] \, \mathrm{turn}_{\mathbf{V}} \tag{5}$$

It means that whatever move is initially made by the falsifier, it will be the verifier's turn. Similarly, at Node 2 (i.e. after the choice of a by the falsifier), the verifier is not ensured to win whatever value she chooses:

$$\mathbf{G}, \mathbf{2} \nVdash [y := a \cup y := b] \, \mathrm{win}_{\mathbf{V}} \tag{6}$$

but she *can* choose one value and win:

$$\mathbf{G}, \mathbf{2} \Vdash \langle y := a \cup y := b \rangle \, \text{win}_{\mathbf{V}} \tag{7}$$

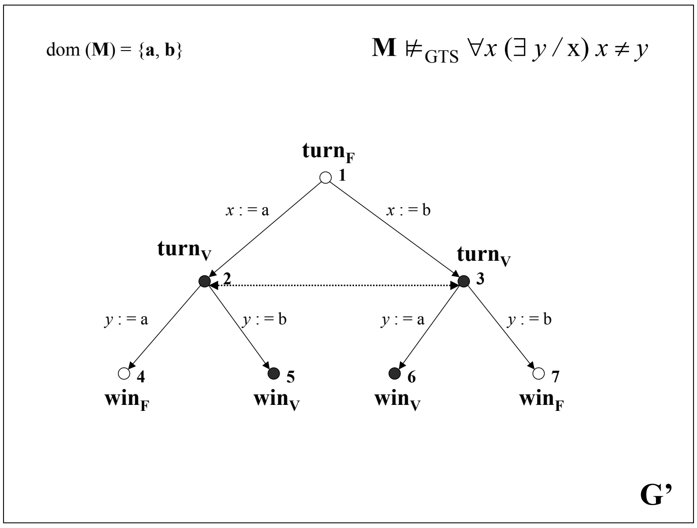
Moreover, one can express that there is a winning strategy for the verifier:

$$\mathbf{G}, \mathbf{1} \Vdash [x := a \cup x := b] \langle y := a \cup y := b \rangle \, \text{win}_{\mathbf{V}} \tag{8}$$

This can be generalized to more complex games, with more complex strategies: the existence of a winning strategy is thus expressed with more complex sequences of action diamonds and boxes.

Example 2 Now, if we introduce games of imperfect information, we can complete the illustration of EAL in a natural way. Consider the evaluation game **G'** of the IF-sentence:

$$\forall x (\exists y / x)(x \neq y) \tag{9}$$



The dotted line indicates the 'information set' for player **V**: it relates two states that are indistinguishable from the verifier's viewpoint. Information sets provide natural candidates for the accessibility relation of the epistemic operator $\text{K}_{\mathbf{V}}$.[2] As van Benthem explains, one can thus see that at state 2 the verifier knows 'de dicto' that she has some winning strategy:

$$\mathbf{G}', \mathbf{2} \Vdash \text{K}_{\mathbf{V}}(\langle y := a \rangle \text{win}_{\mathbf{V}} \vee \langle y := b \rangle \text{win}_{\mathbf{V}}) \tag{10}$$

[2]In fact, one has to consider the complete equivalence relation linking nodes undistinguishable for player V as the required accessibility (or 'alternativeness') relation, i.e. loops would have to be added at each node.

because in every epistemic alternative to 2, namely 2 and 3, she has one:

$$\begin{array}{l} \mathbf{G}', \mathbf{2} \Vdash \langle y := a\rangle \mathrm{win}_{\mathbf{V}} \vee \langle y := b\rangle \mathrm{win}_{\mathbf{V}} \\ \mathbf{G}', \mathbf{3} \Vdash \langle y := a\rangle \mathrm{win}_{\mathbf{V}} \vee \langle y := b\rangle \mathrm{win}_{\mathbf{V}} \end{array} \tag{11}$$

whereas she doesn't know 'de re' which strategy is the winning one:

$$\mathbf{G}', \mathbf{2} \Vdash \neg \mathrm{K}_{\mathbf{V}}\langle y := a\rangle \mathrm{win}_{\mathbf{V}} \wedge \neg \mathrm{K}_{\mathbf{V}}\langle y := b\rangle \mathrm{win}_{\mathbf{V}} \tag{12}$$

The contrast between the two kinds of knowledge for the verifier can hence be accounted for within EAL. Should we stop here and consider that one can get rid of IF languages thanks to this dynamic epistemic logic?

IF-FOL and EAL As a competing frame, has EAL the same expressive power as IF-FOL? As such, the question is meaningless since EAL describes local properties of (evaluation-)games whereas IF-FOL (like standard FOL) describes usual models. To put it in other words: the model-denotation of EAL formulas correspond to the game-denotation of (IF-)FOL formulas. Hence lots of assertions about game trees can be made in EAL that have no counterpart in standard of IF-FOL, such as the following:

- Basic actions are deterministic (which is valid in EAL):
  $\langle \pi \rangle\ \varphi \rightarrow [\pi]\ \varphi$, for all $\pi \in B$.
- Every turn of the falsifier is followed by one of the verifier:
  $\mathrm{turn}_F \rightarrow [\pi]\ \mathrm{turn}_V$, for all $\pi \in B$.
  (or: $\mathrm{turn}_F \rightarrow [\cup_{d \in dom(\mathbf{M})}\ (x{:=}\mathrm{d}) \cup (\mathrm{L} \cup \mathrm{R})]\ \mathrm{turn}_V$)

  and so forth.

Another reason why EAL should not be directly compared to FOL or IF-FOL is the fact that EAL depends on a previously chosen model **M**: basic action modalities such as object picking obviously depend on the domain do(**M**).[3] EAL thus corresponds to an already interpreted language.
So it is interesting to compare the two languages not as wholes, but on specific formulas. I will concentrate on the kind of assertion that play a crucial role in GTS and IF logic, namely the assertion that there is a uniform winning strategy for the initial verifier in $game(\varphi,\ \mathbf{M},\ s)$. Let's denote by $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$ the corresponding EAL formula, when it exists. For instance, if **M** is a two-element model with its domain $\mathrm{do}(\mathbf{M}) = \{a,\ b\}$, the existence of a uniform winning strategy for the verifier in $game((\exists x)\ x = x,\ \mathbf{M},\ s)$ is defined by: $\mathbf{uws}(game((\exists x)\ x = x,\ \mathbf{M},\ s)) = \langle x{:=}a\rangle\ \mathrm{win}_V$.

Unfortunately this easy case does not generalize:

Fact 1 *Standard EAL cannot express that there is a uniform winning strategy for the verifier in* $game(\varphi, \mathbf{M}, s)$.

[3]The usual quantifiers can then be construed as abbreviations for action modalities ($\forall x =_{def} [\cup_{d \in dom(\mathbf{M})} x := d]$; $\exists x =_{def} \langle \cup_{d \in dom(\mathbf{M})} x := d\rangle$).

It is not definable in EAL but in a modal fixed-point extension of EAL (see van Benthem 2000a, 2000b). A demonstration of FACT 1 could consist in showing that for some specific IF-FOL formula φ, the class of games $game(\varphi, \mathbf{M}, s)$ where there is a uniform winning strategy for the verifier is not definable in standard EAL. In the next section, I will only give an illustration of FACT 1 with the example of an IF formula such that the existence of a uniform winning strategy for the verifier in the correlated game is directly definable in an extension of EAL, but with no obvious counterpart in standard EAL.

4 IF Modal Logics and IF-EAL

In comparison with IF-FOL, EAL appears to give a new, more local and fine-grained approach of imperfect-information games. On the other hand, IF-FOL enables to express game-theoretical truth-conditions of FOL formulas, and this cannot be grasped within EAL (see FACT 1). I will now propose a kind of compromise: an extension of EAL which preserves the sharp insight of EAL while increasing its expressive power.

' Slashing' some modal language, i.e. considering its IF version, is one interesting way to extend it. Tulenheimo 2004 is the first systematic work on this issue and it contains several important results. What I will consider here is the 'uniformity interpretation' of the slash-notation for modal languages.[4] This interpretation is grounded in a game-theoretical semantics for modal logics in the same manner as IF-FOL is based on GTS for standard FOL.
Tulenheimo's IF modal logic of k modality types (IFML[k]) is an extension of basic modal logic ML[k] where modal operators are allowed to be independent from specified other modal operators. In other words, formulas such as the following are allowed:

$$[\text{A}]_1\ [\text{A}]_2\ (<\text{C}>/[\text{A}]_2)\ \varphi, \text{ where } \varphi \text{ is a standard ML}[k]\text{-formula}$$

whereas others such as the following, where modalities are independent from connectives, are not:

$$(<\text{A}>_1/\wedge)\ \varphi_1 \wedge (<\text{B}>_2/\wedge)\ \varphi_2$$

The latter is a formula of another logic developed by the same author, *Extended* IF modal logic of k modality types (EIFML[k]).

For the purpose of this paper, I will choose the 'extended' mode of slashing modal logic instead of the restricted mode, since it appears to be easier to handle according to our intuitions about the epistemic operators. However, Tulenheimo demonstrated that IFML[k] is translatable into standard FOL, whereas EIFML[k] is not – it is second-order. One interesting issue would be to check whether the existence of a uniform winning strategy for the verifier is definable in the restricted IF extension of EAL (let's denote it by: IF*-EAL), and

[4]Tulenheimo 2004 proposes two other interpretations, namely the Backwards-Looking Operations interpretation (BLO) and the Algebraic one (ALG).

more generally, what about imperfect-information games cannot be said with IF*-EAL and requires the extended version, IF-EAL.

GTS for (IF-)EAL EAL is a propositional (multi-)modal language: as such, it can have a game-theoretical interpretation. For that purpose, one needs to choose a model before playing. This model is in fact a game. A GTS-interpretation of an EAL-formula leads thus to the construction of a meta-game, a game 'about' the original game. Let's recall that a model for EAL is a tuple:

$$\mathbf{M} = (\mathrm{W},\ \{\mathrm{R}_\pi\}_{\pi\in A},\ \{\sim_{\mathbf{V}},\ \sim_{\mathbf{F}}\},\mathrm{V})$$

where W is a set of states, $\{\mathrm{R}_\pi\}_{\pi\in A}$ the set of accessibility relations corresponding to actions, $\sim_i$ is the accessibility (equivalence) relation for the epistemic operator K_i, and V a valuation function for atomic formulas. (Models for IF-EAL will be the same.)

Now, we can give natural GTS rules for the (basic) action modalities:

- **(G.$\langle\pi\rangle$).** If the game is of the form $game(\langle\pi\rangle\varphi, \mathbf{M}, s)$, then the verifier picks out, if possible, a state t resulting from the execution of π (i.e. $\mathrm{R}_\pi st$); the rest of the game is as in $game(\varphi,\ \mathbf{M},\ t)$; if she cannot choose such a state, she loses and the falsifier wins.
- **(G. $[\pi]$).** $game([\pi]\varphi, \mathbf{M}, s)$ is likewise, except that the falsifier makes the choice.
- **(G. $\mathbf{K_V}$).** If the game is of the form $game(\mathrm{K}_{\mathbf{V}}\varphi,\ \mathbf{M},\ s)$, then the falsifier picks out an epistemic alternative of the current state for the verifier (i.e. a state t s.t. $s \sim_{\mathbf{V}} t$); the rest of the game is as in $game(\varphi,\ \mathbf{M},\ t)$. (*Remark*: As it is assumed that the alternativeness relation is reflexive, the falsifier can always pick out an alternate to the current state.)

We can also add rules for complex action modalities:

- **G**($\langle\cup\rangle$). $game(\langle\pi_1\cup\pi_2\rangle\varphi,\ \mathbf{M},\ s)$ starts with the choice of an index $i \in \{1,2\}$ by the verifier, and the rest of the game is as in $game(\langle\pi_i\rangle\varphi,\ \mathbf{M},\ s)$;
- **G($[\cup]$).** $game([\pi_1\cup\pi_2]\varphi,\ \mathbf{M},\ s)$ is likewise, except that the falsifier makes the choice;
- **G($\langle;\rangle$).** $game(\langle\pi_1;\pi_2\rangle\ \varphi,\ \mathbf{M},\ s)$ is like $game(\langle\pi_1\rangle\langle\pi_2\rangle\varphi,\ \mathbf{M},\ s)$

So we get, for any IF-EAL formula φ:

$\mathbf{M}, s \Vdash_{\mathrm{GTS}} \varphi$ $\quad\Leftrightarrow\quad$ there is a winning strategy for the verifier in $game(\varphi, \mathbf{M}, s)$.

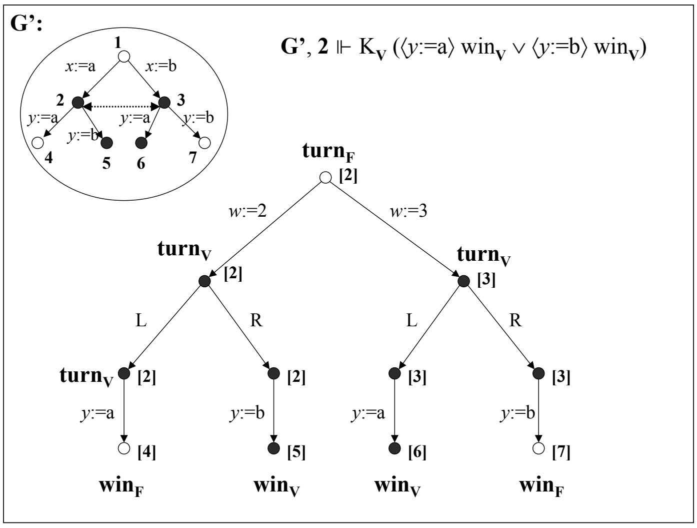
IF-Epistemic Action Logic Game-theoretically interpreted, EAL can be extended to cases of imperfect-information and lead to Independence-Friendly Epistemic Action Logic (IF-EAL). The motivation for such an extension rests on the ability of IF epistemic logic to account for the distinction between knowledge de dicto and knowledge de re (*knowing-that* vs. *knowing-what*, *who*, *which*... in Hintikka's terminology). As we have already seen, in standard EAL one can express the knowledge de dicto of the winning strategy by means of a propositional disjunction

$$\mathbf{G}', \mathbf{2} \Vdash \mathrm{K}_{\mathbf{V}}(\langle y := a\rangle \mathrm{win}_{\mathbf{V}} \vee \langle y := b\rangle \mathrm{win}_{\mathbf{V}}) \quad (10)$$

or by means of some complex action modality involving the union symbol:

$$\mathbf{G}', \mathbf{2} \Vdash \mathrm{K}_{\mathbf{V}}(\langle y := a \cup y := b\rangle \mathrm{win}_{\mathbf{V}}) \quad (13)$$

We can check the extensive game for (10):



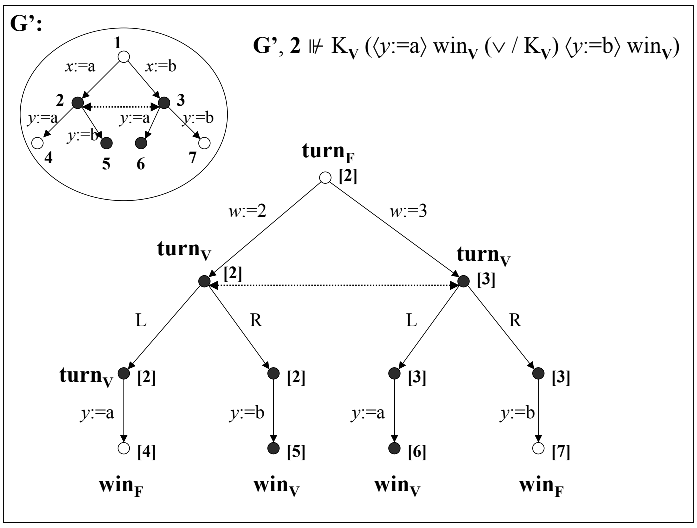
Then the ignorance de re of the same winning strategy can be accounted for in IF-EAL with the slash notation, applied either to the disjunction:

$$\mathbf{G}', \mathbf{2} \nVdash \mathrm{K}_{\mathbf{V}}(\langle y := a\rangle \mathrm{win}_{\mathbf{V}}(\vee/\mathrm{K}_{\mathbf{V}})\langle y := b\rangle \mathrm{win}_{\mathbf{V}}) \quad (14)$$

or to the union symbol:

$$\mathbf{G}', \mathbf{2} \nVdash \mathrm{K}_{\mathbf{V}}(\langle y := a(\cup/\mathrm{K}_{\mathbf{V}})y := b\rangle \mathrm{win}_{\mathbf{V}}) \quad (15)$$

(As was announced before, these formulas clearly belong to the *extended* IF version of EAL.) Both formulas (14) and (15) mean that the choice of picking a or picking b is independent from the knowledge of the verifier. (Formula (15) indicates a new kind of complex actions, whose status is not clear at first



glance!). In order to evaluate the IF-EAL sentences using GTS, we need not introduce new rules: the rules for standard EAL can do the job.

Generally speaking, in order to obtain IF-EAL we have to allow several patterns of independence between operators – and in fact, lots of them – together with the correlated formula forms :

- $(\langle\pi_2\rangle/[\pi_1])$, $(\langle\pi\rangle/\mathrm{K}_{\mathbf{V}})\ldots$
- $(\vee/[\pi])$, $(\vee/\mathrm{K}_{\mathbf{V}})$, $(\vee/[\pi],\mathrm{K}_{\mathbf{V}})$

and lots of new (complex action) modalities:

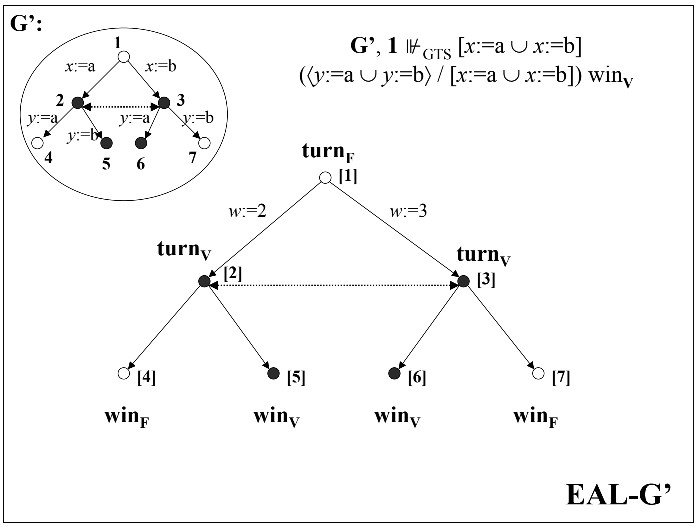
- $\langle\pi_1\ (\cup/[\pi])\ \pi_2\rangle$, $\langle\pi_1\ (\cup/\mathrm{K}_{\mathbf{V}})\ \pi_2\rangle$, $\langle\pi_1\ (\cup/[\pi],\ \mathrm{K}_{\mathbf{V}})\ \pi_2\rangle\ldots$
- $[\pi_1(\cup/[\pi])\ \pi_2]$, $[\pi_1(\cup/\langle\pi\rangle)\ \pi_2]$, $[\pi_1(\cup/\mathrm{K}_{\mathbf{V}})\ \pi_2]\ldots$

IF-EAL thus seems to provide a good account of different kinds of verifier's knowledge through the game process. Now, as our main concern is evaluation games we have to look at what comes about at the root, that is, at Node 1 of the initial game.

At the root of game **G'**, there is no uniform winning strategy for the verifier: this can be expressed with each of the following equivalent formulas:

$$\begin{array}{l} \mathbf{G'}, \mathbf{1} \nVdash_{\mathrm{GTS}} [x := a \cup x := b](\langle y := a \cup y := b\rangle/[x := a \cup x := b])\mathrm{win}_{\mathbf{V}} \\ \mathbf{G'}, \mathbf{1} \nVdash_{\mathrm{GTS}} [x := a \cup x := b](\langle y := a(\cup/[x := a \cup x := b])y := b\rangle \mathrm{win}_{\mathbf{V}} \end{array} \tag{16}$$

But in fact, the following formula which states that there is a winning strategy for the verifier in the corresponding perfect information game **G**, still holds in **G'**:



$$\mathbf{G}', \mathbf{1} \Vdash_{\mathrm{GTS}} [x := a \cup x := b]\langle y := a \cup y := b\rangle \mathrm{win}_{\mathbf{V}} \tag{17}$$

Indeed, there is still a winning strategy for the verifier in the imperfect information game (17), but it is not a uniform one (16). Or to put it in other words: the verifier still has a winning strategy, but it is no more available to her. The contrast between the EAL formula in (17) ("there is a winning strategy") and the IF-EAL formula in (16) ("there is no uniform winning strategy") constitutes an interesting illustration of FACT 1: no obvious standard EAL formula appears that would do the job of the IF-EAL formula about the uniform strategy.

5 Game Comparison

Isomorphism One can compare the evaluation game **G'** of our original IF-sentence (6):

$$\forall x(\exists y/x)(x \neq y) \tag{6}$$

in the model **M**, with the evaluation game **EAL-G'** of the assertion of the existence of some uniform winning strategy for the verifier in game **G**:

$$\mathbf{M} \nvDash_{\mathrm{GTS}} \forall x(\exists y/x)(x \neq y) \tag{18}$$

$$\mathbf{G}', \mathbf{1} \nVdash_{\mathrm{GTS}} [x := a \cup x := b](\langle y := a(\cup/[x := a \cup x := b])y := b\rangle \mathrm{win}_{\mathbf{V}} \tag{16}$$

It's worth noting the following: There is an obvious *bisimulation* between G' and EAL-G' relating the roots. Consequently: *The roots of the games G' and EAL-G' verify the same EAL formulas* (van Benthem 2000b, 162).

But this result is still limited: the roots of the 'object-game' and of the 'meta-game' verify the same *standard* EAL formulas, and of course this does not mean that they share every *IF*-EAL formula. As IF multi-modal logic is strictly more expressive than the corresponding standard fragment (see Tulenheimo 2004), the bisimulation relating the roots of the two games is not enough to ensure that the roots verify the same IF-EAL formulas, especially those stating that there's a winning strategy for the verifier in the evaluation of an IF first-order formula. Therefore, in order to extend the equivalence of some first-order sentence with its epistemic GTS-oriented form to *IF* first-order sentences, we need more than bisimulation.

Fortunately we have here a higher, and in fact the highest degree of similarity between the two games, namely *isomorphism*, and this can easily be generalized to other (IF or standard) first-order sentences φ:

Fact 2 *At the root of $G = game(\varphi, \mathbf{M}, s)$, the evaluation game of $\mathbf{uws}(G)$ is isomorphic to the original game G:*

$$game(\mathbf{uws}(G), G, \mathbf{root}) \cong G. \tag{2}$$

Indeed, let's consider such a formula φ in prenex normal form, and its transformation $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$ into a IF-EAL formula, stating that there is a uniform winning strategy for the verifier in the game associated with φ:

(i) Replace the (independent) connectives by quantifiers, e.g.: $\forall x\ (\psi_1\ (\vee/\forall x)\ \psi_2)$ will be transformed into: $\forall x\ (\exists i/\forall x)\ \psi(i)$, where $\psi(i) = \psi_i$.

(ii) φ is now of the form $\mathrm{Q}^0 x^0 (\mathrm{Q}^1 x^1/\mathrm{W}^1)(\mathrm{Q}^2 x^2/\mathrm{W}^2)\ldots(\mathrm{Q}^n x^n/\mathrm{W}^n)\ \psi$, where Q^i is a quantifier, W^i the set of quantifiers Q^i is independent from ($\mathrm{W}^i \subseteq \{\mathrm{Q}^0, \ldots, \mathrm{Q}^{i-1}\}$), and ψ the matrix. Each quantifier $\mathrm{Q}^i x^i$ can be translated in the following way:

- if it is a universal quantifier ($\forall^i x^i$), then replace it by the "box": $[\cup^i{}_{d\in dom(\mathbf{M})}\ (x^i := \mathrm{d})]$,
- if it is an existential one ($\exists^i \mathrm{x}^i$), then replace it by the "diamond": $\langle \cup^i{}_{d\in dom(\mathbf{M})}\ (x^i := \mathrm{d})\rangle$.

Such a translation is to be effected also for quantifiers in the sets W^i.

(iii) Replace the matrix ψ by $\mathrm{win}_{\mathbf{V}}$.

For instance, from the first-order sentence: $\forall^0 x^0 (\exists^1 x^1/\forall^0 x^0)\,\psi(x^0, x^1)$ on a model with two elements ($\mathrm{dom}(\mathbf{M}) = \{a,\ b\}$), we will reach the IF-EAL formula:

$$[(x^0 := a) \cup^0 (x^0 := b)](\langle (x^1 := a) \cup^1 (x^1 := b)\rangle/[(x^0 := a) \cup^0 (x^0 := b)])\,\mathrm{win}_{\mathbf{V}} \tag{16}$$

According to this transformation, the evaluation game of a first-order sentence φ and that of the corresponding formula $\mathbf{uws}(game(\varphi,\ \mathbf{M},\ s))$ are obviously isomorphic: this could be proved by a straightforward induction on the complexity of (the prefix of) φ.

Stop the regression As the evaluation game of φ, $game(\varphi, \mathbf{M}, s)$, and its meta-game (i.e. the evaluation game of $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$) are isomorphic, their roots verify the same *IF*-EAL formulas. It leads to the following fact:

Fact 3 *$G = game(\varphi, \mathbf{M}, s)$ is enough – i.e. in order to see whether the verifier has a uniform winning strategy in $game(\mathbf{uws}(G), G, \mathbf{root})$, no more 'meta game' is needed.*

This explains how to stop the headlong rush apparently threatening the whole enterprise: While extending EAL (which was designed to escape from IF) into an IF version, we are not led to build a new language to speak about the new games. IF-EAL is enough: $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$ does not only state that there is a winning strategy for the verifier in the evaluation game of φ, $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$ also states that there is a winning strategy for the verifier *in its own evaluation game.*

6 Discussion

Some advantages of IF-EAL Our IF extension of EAL is not superfluous, as EAL is expected to account for the winning strategies of the evaluation games (among other things). Thanks to informational independence, the enriched version of EAL can account for imperfect information evaluation games in a straightforward way.

1. IF-EAL enables to formulate the contrast between knowledge de dicto and ignorance de re in a way which is more natural than standard EAL: this can be seen with the EAL formula (12), that is literally rendered by: "(At Node 2) the verifier doesn't know whether choosing a is a winning strategy, and she doesn't know whether choosing b is a winning strategy." By contrast, formula (15) is directly read as: "(At Node 2) the verifier doesn't know which choice is a winning strategy". And the gap between (10) and (12) – expressing the difference between knowledge de dicto and knowledge de re – should similarly be compared to the distinction between (10) and (14) (or between (13) and (15)).

2. As was already mentioned, what formulas (16)-(17) reveal is that the non-existence of uniform winning strategy for the verifier in the whole game is not expressible in a direct way in standard EAL. And we will see below that the *knowledge* of the verifier is of no help in such cases.

3. Some IF-EAL formulas cannot be translated into standard EAL formulas. An example is provided by the following schema:

$$[\,]_1 \mathrm{K}_{\mathbf{V}} [\,]_2 (\langle\,\rangle / \mathrm{K}_{\mathbf{V}})\, \varphi \tag{17}$$

where the diamond is independent from the epistemic operator, but still dependent from the boxes. One could meet such a schema in the evaluation game of e.g.:

$$\forall x \forall y \exists z (x + y = z) \tag{18}$$

stating that whatever value is chosen for x by the falsifier, the verifier will know *de re* what is her winning strategy – and this is certainly true!

Epistemic statements Let's go back to what happens with the epistemic operator at the root of the games. Relatively to the alternativeness relation ($\sim_{\mathbf{V}}$), evaluation games of IF first-order sentences start with a reflexive singleton – because informational independence (namely, independent quantifiers) cannot occur just at the beginning of a sentence, but only 'inside' it.

As a result, we have the following implication and equivalence for any formula of IF-EAL:

$$\mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \varphi \quad \Rightarrow \quad \mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathrm{K}_{\mathbf{V}}\varphi \tag{19}$$

$$\mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathrm{K}_{\mathbf{V}}\varphi \quad \Leftrightarrow \quad \mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathrm{K}_{\mathbf{V}}(\varphi/\mathrm{K}_{\mathbf{V}}) \tag{20}$$

where $(\varphi/\mathrm{K}_{\mathbf{V}})$ is the IF-EAL sentence resulting from φ by the replacement of each action diamond $(\langle\pi\rangle/\mathrm{W})$ by the $\mathrm{K}_{\mathbf{V}}$-liberated corresponding one, $(\langle\pi\rangle/\mathrm{W}, \mathrm{K}_{\mathbf{V}})$, and the same for each disjunction $(\vee/\mathrm{W})$. Moreover as the alternativeness relation is reflexive, the knowledge property $(\mathrm{K}_{\mathbf{V}}\varphi \rightarrow \varphi)$ holds in our frame: the implication (19) actually leads to an equivalence:

$$\mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \varphi \quad \Leftrightarrow \quad \mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathrm{K}_{\mathbf{V}}\varphi \tag{21}$$

and, combined with (20), we obtain an interesting equivalence between any IF-EAL formula and its 'epistemic' version:

$$\mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \varphi \quad \Leftrightarrow \quad \mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathrm{K}_{\mathbf{V}}(\varphi/\mathrm{K}_{\mathbf{V}}) \tag{22}$$

Hence at the root, *the epistemic operator cannot provide any new and interesting description* of the game. However we can raise an interesting question with this result: In what sense can a first-order sentence φ be said equivalent to what I shall call its *epistemic game-oriented form*, i.e. to the IF epistemic formula asserting the knowledge de re of a winning strategy by the verifier in the evaluation-game of the original sentence? Let's denote by $\mathbf{egof}(\varphi)$ the *epistemic game-oriented form* of φ: $\mathbf{egof}(\varphi)$ belongs to IF-FOEL (IF first-order epistemic logic); it is like $\mathrm{K}_{\mathbf{V}}(\varphi/\mathrm{K}_{\mathbf{V}})$, where $(\varphi/\mathrm{K}_{\mathbf{V}})$ is the IF sentence resulting from φ by the replacement of each existential quantifier $(\exists^i x^i/\mathrm{W}^i)$ by the $\mathrm{K}_{\mathbf{V}}$-liberated corresponding one, $(\exists^i x^i/\mathrm{W}^i, \mathrm{K}_{\mathbf{V}})$. For instance:

$$\mathbf{egof}(\forall x \exists y (x \neq y)) = \mathrm{K}_{\mathbf{V}} \forall x (\exists y/\mathrm{K}_{\mathbf{V}})(x \neq y) \tag{23}$$

We can now compare the respective 'translations' of φ and $\mathbf{egof}(\varphi)$ into IF-EAL, i.e. respectively $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$ and $\mathbf{uws}(game(\mathbf{egof}(\varphi), \mathbf{M}, s))$, stating that there is a uniform winning strategy for the verifier in the game associated with φ, $\mathbf{M}$, s, and the same with $\mathbf{egof}(\varphi)$. It is easily seen that in IF-EAL, $\mathbf{uws}(game(\mathbf{egof}(\varphi), \mathbf{M}, s)) = \mathrm{K}_{\mathbf{V}}(\mathbf{uws}(game(\varphi, \mathbf{M}, s))/\mathrm{K}_{\mathbf{V}})$. Consequently, and according to (22), for every IF first-order formula φ ($\mathbf{G}$ being isomorphic to its evaluation game):

$$\mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathbf{uws}(game(\varphi, \mathbf{M}, s)) \quad \Leftrightarrow \quad \mathbf{G}, \mathbf{1} \Vdash_{\mathrm{GTS}} \mathbf{uws}(game(\mathbf{egof}(\varphi), \mathbf{M}, s)) \tag{24}$$

This means that there is a uniform winning strategy for the verifier in the game associated with a specific formula φ if and only if there is one in the game associated with the 'epistemic game-oriented form' of φ. Now, the right side of the equivalence, $\mathbf{uws}(game(\mathbf{egof}(\varphi), \mathbf{M}, s))$ can be read in the following two ways: (i) it can mean that the verifier in $game(\varphi, \mathbf{M}, s)$ knows (de re) which is the winning strategy for herself (this is the reason why it is equivalent to $\mathbf{ws}(\varphi)$); (ii) it can also be understood as meaning that there is a winning strategy for $\mathbf{egof}(\varphi)$ in the evaluation game $game(\varphi, \mathbf{M}, s)$ of φ in $\mathbf{M}$ (which leads to the intended equivalence). So (24) exactly states that φ is GTS-true iff $\mathbf{egof}(\varphi)$ is GTS-true: this is the expected equivalence.

To sum up: any IF first-order formula φ is equivalent to a correlated formula whose meaning is "The verifier (of the evaluation game of φ) *knows* de re *which* strategy is a winning strategy for herself". This reflexive feature of IF logic, usually claimed in an informal way, can be established within the EAL frame which – against Hintikka – takes evaluation games and their players' knowledge and powers at face value. However, this result is established thanks to the application of two Hintikkian ideas to EAL: IF extension, and the epistemic concept of *knowing-wh.*

Is it Genuine Knowledge? The equivalence (20) given above implies that there will be no more distinction between the verifier's knowledge 'de re' of her (uniform) winning strategy, and her corresponding knowledge 'de dicto'. This would threaten the whole construction of our epistemic logic, if it were to hold in general, but here, of course, it is not the case (as it can be seen e.g. at Node 2 in the preceding examples). In fact, this equivalence can be read in a more 'positive' way: (20) means that the verifier's knowledge of the existence of some winning strategy implies her knowledge of that strategy. If the frame employed here is a suitable one, it means that the verifier in evaluation games is a '*perfect knower*' in some sense. This meets the requirement that players of such games be ideal players. What is more: The equivalence (24) between a sentence and its epistemic game-oriented form strongly reinforces the idea that the truth of a sentence is a property of the 'game-board' rather than of the game course.

However, we have reached an interesting phenomenon with IF-EAL. The sentences designed to describe the evaluation game of (IF or standard) first-order sentences describe their own evaluation games. This reflexive feature is in fact independent from any 'epistemic' property of the players: what we needed to arrive at it is only dynamic logic, with no epistemic operator.

7 Conclusion

Dealing with imperfect-information games, we are usually faced with two competing frames: IF-FOL and EAL. After having observed that the two logics provide complementary views on games, I proposed to consider an extension of EAL: IF-EAL, based on some game-theoretical semantics for dynamic logic. Thanks to this new IF multi-modal, dynamic and epistemic language, we can express for any IF-FOL formula φ the existence of a uniform winning strategy for the verifier of some corresponding evaluation game with a formula $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$ which does not belong to standard EAL. In general, different epistemic assertions about the players appear to be more intuitive in the extended version than in the original one.

Moreover, we showed that $\mathbf{uws}(game(\varphi, \mathbf{M}, s))$ *constitutes its own truth-conditions*, since it coincides with the assertion of the existence of some winning strategy in its own GTS evaluation game. This is an EAL-correlate of a well-known 'reflexive fact' in IF-FOL, namely that the truth-conditions of a formula can be formulated in the same language, using the very same formula. Another correlate of the same equivalence was established in IF-FOEL, φ being equivalent to $\mathbf{egof}(\varphi)$, its epistemic game-oriented form. Finally, asserting a formula and asserting that the initial verifier knows which is the winning strategy in its evaluation game, are the same assertion.

The important fact about these equivalences which all reflect Hintikka's idea that IF languages can define their own truth predicate, is that it can stop the indefinite regression IF/EAL/IF/EAL... Van Benthem indeed created EAL to escape from IF logic. Taking evaluation games seriously, EAL gives a local and precise perspective on features of games that were neglected from the global viewpoint of FOL, IF or not. What is more: EAL reduces informational independence to dynamic and epistemic features of players of evaluation games. Then 'slashing' EAL seems going back to the prior situation.

However this procedure is not worthless. IF-EAL formulas asserting the existence of a uniform winning strategy in a game G have the nice property that their own evaluation game is G. They are simultaneously about G, and evaluated by G. The hierarchy of games and meta-games thus stops with IF-EAL formulas.

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