ABOUT GAMES AND SUBSTITUTION

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ABSTRACT

Kripke's substitutional interpretation of quantifiers is usually said to be unsatisfactory for independence-friendly (IF) languages. The purpose of this paper is to question this claim. Two accounts of substitutional semantics for IF sentences will be written down, and the objection of the so-called 'dummy variables' will be ruled out. Moreover, it will be argued, against the traditional view, that Game-Theoretical Semantics (GTS) should be conceived of as substitutional. The paper ends with some remarks concerning the reasons why substitution is especially suitable for dynamic semantics.

1. INTRODUCTION

In spite of its defense by Kripke (1976), the substitutional interpretation of quantifiers remained marginal and neglected by the logical tradition as an exotic and uninteresting view. However, it seems that the so-called 'dynamic turn' in natural language semantics should provide a second chance for substitution. Why? Because dealing with dynamic phenomena such as anaphoric relations, what is relevant to the processing of a pronoun is not directly the object involved as the antecedent but its name or description; and while standard (Tarski-like) semantic theories focus on the objects of a domain, substitutional accounts of quantifiers concentrate on their individual designators. The substitutional (or syntactical) conception of quantifiers thus seems better suited for dynamic phenomena than the objectual one. (I will return to this issue in Section 7.)

On the other hand, Game-Theoretical Semantics (GTS) claims to account for dynamic phenomena such as anaphora resolution, whereas it is usually assumed to be a kind of objectual interpretation. This objectual approach carries over to the informationally-independent quantifiers which naturally emerge from game semantics. Indeed, Hintikka and Sandu explicitly argue that substitution cannot account for independence-friendly (IF) sentences, and must therefore be ruled out.

Is a substitutional interpretation of IF languages impossible? More: is GTS really an objectual semantics? These are some of the questions we will be concerned with.

2. About the Alleged Incompatibility of Substitution with IF Logic

Let's start with the issue whether IF logic and Kripke's substitutional interpretation of quantifiers are compatible, or not.

Kripke's proposal goes as follows: He first considers a formalized language L_0, where truth is already granted, *then* an extension L_1 of this language which introduces new, substitutional variables (x_1, x_2 …) and substitutional quantifiers (Σ and Π). For that purpose, a special class of expressions of L_0 must be recognized, the *substitution class* C, whose elements are called *terms*. We only assume that this class is not empty, but we don't have to assume that *terms* are to 'denote' any object. Kripke says that: "Terms could be any class of expression of L_0, sentences, connectives, even parentheses" (1976: 329).

Truth can then be defined for sentences of L_1. We define the truth of existentially quantified sentences $(\Sigma x_i)\phi$ as the existence of at least one true substitution-instance of the subformula ϕ; in the same manner, we can characterize the truth of universally quantified sentences $(\Pi x_i)\phi$ as the truth of every substitution-instance of ϕ – the propositional connectives are defined in the standard way:

(1). $\neg\phi$ is true iff ϕ is not;
(2). $\phi\wedge\psi$ is true iff ϕ is and ψ is;

(3). $(\Sigma x_i)\phi$ is true iff there is a term $a \in C$ such that $[a/x_i]\phi$ is true;

(4). $(\Pi x_i)\phi$ is true iff for every term $a \in C$, $[a/x_i]\phi$ is true

(where $[a/x_i]\phi$ comes from ϕ by replacing all free occurrences of x_i by a).

Is Kripke's account of quantifiers adequate for IF languages? According to Hintikka and Sandu, it is not. They put forward a counter-example, that is an example of two IF sentences that should – so they say – have the same truth-values according to the substitutional interpretation, whereas the game-theoretical interpretation clearly indicates it is not the case:

> "… ultimate quantifier-free substitution-instances of [HS-1] and [HS-2]
>
> [HS-1] $(\forall x)\,(\forall z)\,(\exists y)\,(\exists u)\,S[x, y, z, u]$ and
>
> [HS-2] $(\forall x)\,(\forall z)\,(\exists y/\forall z)\,(\exists u/\forall x)\,S[x, y, z, u]$
>
> are the same. Hence, if the substitutional interpretation view is correct, [HS-1] and [HS-2] should have the same truth-values. But it is a well-known result that [HS-2], which under the game-theoretical interpretation assumed here is logically equivalent to
>
> [HS-3] $(\exists f)\,(\exists g)\,(\forall x)\,(\forall z)\,S[x, f(x), z, g(z)]$,
>
> has no first-order equivalent. Hence, the substitutional account of quantifiers does not work for IF-first order logic." (Hintikka and Sandu, 1994: 121)

Is it really a counter-example? This is dubious. Of course, Kripke's theory cannot directly do the job, because it has not been built up to account for IF languages. But an extension of Kripke's substitutional interpretation can be constructed, which will allow to draw a distinction between the two sentences.

3. First Strategy: Substitution in the Metalanguage

Kripke didn't restrict his construction of quantifiers Σ and Π so that they should replace the objectual or 'referential' ones, $\forall$ and $\exists$. As already noted, Σ and Π need not operate on the equivalent of objectual individual variables, they can operate on *any* expression of the basic language L_0. Kripke even contemplates the possibility of constructing a substitutional account of quantifiers for the metalanguage through a new extension of the language that would involve new quantifiers "whose substitutes are sentences of the language [L_1]" (1976: 368).

So we can, from a basic language L_0, where the truth-values of atomic sentences are defined, construct a first extension L_1 corresponding to a classical first-order language, next a second one L_2, corresponding to a classical second-order language. We thus obtain a two-step strategy:

L_0: basic language

Logical constants: $\wedge$, $\neg$, $=$.

No variable.

Signature: $S_0 = C_1 \cup C_2 \cup C_3$

$C_1 = \{a, a_1, a_2, \ldots\}$	constant symbols
$C_2 = \{f, f_1, f_2, \ldots\}$	function symbols
$C_3 = \{P_1, P_2, \ldots\}$	relation symbols

Sentences of L_0 are *basic sentences*.[1]

L_1: first extension of L_0

Variables of L_1: $x, x_1, x_2, \ldots$

Substitution class: C_1.

Substitutional quantifiers: Σ_1, Π_1.

$(\Sigma_1 x_i)\phi$ is true iff there is a term $a_j \in C_1$ such that $[a_j/x_i]\phi$ is true.

$(\Pi_1 x_i)\phi$ is true iff for every term $a_j \in C_1$, $[a_j/x_i]\phi$ is true.

L_2: second extension of L_0

Variables of L_2: $X, X_1, X_2, \ldots$

Substitution class: C_2.

Substitutional quantifiers: Σ_2, Π_2.

$(\Sigma_2 X_i)\phi$ is true iff there is a term $f_j \in C_2$ such that $[f_j/X_i]\phi$ is true.

$(\Pi_2 X_i)\phi$ is true iff for every term $f_j \in C_2$, $[f_j/X_i]\phi$ is true.

We can now reformulate Hintikka and Sandu's second-order equivalent (HS-3) of the IF first-order sentence (HS-2) as follows:

(L_2-3). $$(\Sigma_2 X_1)(\Sigma_2 X_2)(\Pi_1 x_1)(\Pi_1 x_2)\, S[x_1, X_1(x_1), x_2, X_2(x_2)]$$

which is equivalent to

(L_2-3*). $(\Sigma_2 X_1)\ (\Sigma_2\ X_2)\ \phi(X_1,X_2)$,

where $\phi(X_1,X_2) = (\Pi_1 x_1)\ (\Pi_1 x_2)\ S[x_1, X_1(x_1), x_2, X_2(x_2)]$

(L_2-3*) is true in L_2 iff there are two terms f_j and f_k in C_2 such that $\phi(f_j, f_k)$ is true ($\phi(f_j, f_k)$ is a formula of L_1):

(L_1-3*) $\phi(f_j, f_k) = (\Pi_1 x_1)\ (\Pi_1 x_2)\ S[x_1, f_j(x_1), x_2, f_k(x_2)]$

Then (L_1-3*) is true in L_1 iff for every term a_m and a_n in C_1, the atomic formula (L_0-3*) is true:

(L_0-3*) $S[a_m, f_j(a_m), a_n, f_k(a_n)]$

So, to recapitulate, according to this interpretation, the original IF sentence (HS-2) is true iff there are two terms f_j and f_k in C_2 such that for every pair of terms a_m and a_n in C_1, the atomic formula (L_0-3*) is true.

By contrast, sentence (HS-1) does not yield the same interpretation, because we do not need to add a second extension. We can express an equivalent of (HS-1) in our first extension L_1:

(L_1-1) $(\Pi_1 x_1)\ (\Pi_1 x_2)\ (\Sigma_1 x_3)\ (\Sigma_1 x_4)\ S[x_1, x_2, x_3, x_4]$

which is true iff for every pair of terms a_j and a_k in C_1 there are two terms a_m and a_n in C_1 such that $S[a_j, a_k, a_m, a_n]$ is true.

As a result, it seems that Kripke's substitutional interpretation of quantifiers can achieve a suitable semantics for IF languages. One would simply have to add a recursion clause for independent quantification, such as:

(IF-L_1-Σ) $(\Pi_1 x_1)\ldots(\Pi_1 x_k)\ (\Sigma_1 x/\Pi_1 x_{i1},\ldots \Pi_1 x_{im})\ S[x, x_1, \ldots x_k]$ is true
iff there is a term f in C_2 such that
$(\Pi_1 x_1)\ldots(\Pi_1 x_k)\ S[f(x_{j1}, \ldots x_{jn}), x_1, \ldots x_k]$ is true
where $\{x_{i1},\ldots x_{im}\}$ is a subset of $\{x_1, \ldots x_k\}$,
and $\{x_{j1}, \ldots x_{jn}\} = \{x_1, \ldots x_k\}\backslash\{x_{i1},\ldots x_{im}\}$.

Unfortunately, this schema is only working under very strong conditions. Because substitution-instances cannot be more than denumerable, the substitution classes are to be at most denumerable. This implies that the terms of L_2, that is the function symbols, be at most denumerable too, and the latter fact itself implies that there are only finitely many constant symbols. So, although this construction covers lots of everyday practices, which deal with finite universes of individuals, it cannot claim to account for IF languages in general.

4. SECOND STRATEGY: SUBSTITUTION IN GTS

Consequently, we have to look for another strategy. I will now propose a mixed one, a kind of 'substitutional GTS'. Recall Hintikka's GTS rules:

> The entire game is played on some given model **M** of the underlying language…
> (R.At). If *A* is a true atomic sentence (or identity), the verifier wins G(*A*) and the falsifier loses it. If *A* is a false atomic sentence (or identity), vice versa.
> (R.$\vee$). G($S_1 \vee S_2$) begins with the choice by the verifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).
> (R.$\wedge$). G($S_1 \wedge S_2$) begins with the choice by the falsifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).
> (R.$\exists$). G(($\exists x$)$S[x]$) begins with the choice by the verifier of a member of do(**M**). If the name of the individual is *b*, the rest of the game is as in G($S[b]$).
> (R.$\forall$). G(($\forall x$)$S[x]$) is likewise, except that the falsifier makes the choice.
> (R.~). G(~*S*) is like G(*S*), except that the roles of the two players (as defined by these rules) are interchanged.

GTS is built as an extension of an objectual account of atoms. In other words, the semantic analysis according to GTS takes complex formulas as inputs, and stops when atomic formulas are reached: the classical Tarski-type semantics thus takes over, as one can see in the atomic-rule, (R.At).

GTS is thus built as an extension of the restriction of Tarskian semantics to atomic formulas. A mixed solution would construct a double-step extension: starting with some basic language L_0 of variable-free formulas, a first extension L_1 would introduce classical substitutional quantifiers, then a second extension L_{IF} would add the slash notation. Hence GTS would account for IF formulas which are not classical, until a classical formula is reached; then Kripke's semantics will take over:

L_0: basic language

Logical constants: $\wedge$, $\neg$, =.

No variable.

Signature: $S_0 = C_1 \cup C_2 \cup C_3$

$C_1 = \{a, a_1, a_2, \ldots\}$ constant symbols

$C_2 = \{f, f_1, f_2, \ldots\}$ function symbols

$C_3 = \{P_1, P_2, \ldots\}$ relation symbols

Sentences of L_0 are *basic sentences*.

L_1: first extension of L_0

Variables of L_1: x, x_1, x_2, …

Substitution class: C_1.

Substitutional quantifiers: Σ_1, Π_1.

$(\Sigma_1 x_i)\phi$ is true iff there is a term $a_j \in C_1$ such that $[a_j/x_i]\phi$ is true.

$(\Pi_1 x_i)\phi$ is true iff for every term $a_j \in C_1$, $[a_j/x_i]\phi$ is true.

L_{IF}: second extension of L_0

Introduction of the slash notation: /.

We thus obtain a new formulation of the GTS rules:

Mixed game-theoretical semantic rules for L_{IF}

The entire game is played on some given interpretation of L_1, with a substitution class of constant symbols C_1.

(R.L_1). If ϕ is a formula of L_1, then the verifier wins G(ϕ) if ϕ is true, and the falsifier wins if ϕ is false.

(R.GTS). If ϕ is a formula of $L_{IF} \backslash L_1$, then rules (R.$\wedge$), (R.$\vee$), (R.Σ), (R.Π) and (R.~) apply.

(R.$\vee$). G($S_1 \vee S_2$) begins with the choice by the verifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).

(R.$\wedge$). G($S_1 \wedge S_2$) begins with the choice by the falsifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).

(R.S). G((Σx)$S[x]$) begins with the choice by the verifier of a member a_i of the substitution class C_1. The rest of the game is as in G($S[a_i]$).

(R.P). G((Πx)$S[x]$) is likewise, except that the falsifier makes the choice.

(R.~). G(~S) is like G(S), except that the roles of the two players (as defined by these rules) are interchanged.

Let's consider Hintikka and Sandu's pair of examples again:

(HS-1) $(\forall x)\ (\forall z)\ (\exists y)\ (\exists u)\ S[x, y, z, u]$
(HS-2) $(\forall x)\ (\forall z)\ (\exists y/\forall z)\ (\exists u/\forall x)\ S[x, y, z, u]$

(HS-1) is a classical formula, i.e. a L_1-sentence; so it is true iff there is a winning strategy for the initial verifier in the correlated game, that is, according to (R.L_1), iff it is true in L_1; one has then to apply the substitutional truth-definition of L_1-sentences: (HS-1) is true iff for every pair of terms a_j and a_k in C_1 there are two terms a_m and a_n in C_1 such that $S[a_j, a_k, a_m, a_n]$ is true in L_0. (We thus obtain the same result as in the first strategy.)

(HS-2) is a non-classical IF-sentence: it is true iff there is a winning strategy for the initial verifier, that is, according to (R.GTS), iff the verifier can: (i) choose a constant a_m in C_1 for y independently of the previous choice of a constant a_k for z by the falsifier, and (ii) choose a constant a_n in C_1 for u independently of the previous choice of a constant a_j for x by the falsifier, such that $S[a_j, a_k, a_m, a_n]$ is true in L_0.

In order to observe the effective combination of game-theoretical and substitutional semantics, let us consider for instance the following formula:

$$\phi = (\Pi x)(\Sigma y/\Pi x)(\Sigma z)\ P[x, y, z]$$

ϕ is true in some interpretation of L_1 iff there is a winning strategy for the initial verifier, that is, iff the verifier can choose a constant a_i for y independently of the previous choice of a constant a_j for x by the falsifier, such that the L_1-sentence $(\Sigma z)\ P[a_j, a_i, z]$ is true; then $(\Sigma z)\ P[a_j, a_i, z]$ is true in L_1 iff there is a constant a_k in C_1 such that $P[a_j, a_i, a_k]$ is true in L_0.

The latter example straightforwardly suggests a refinement of our mixed semantics: wouldn't it be more natural to define the truth of L_1-sentences in a game-theoretical way?

As a matter of fact, we can get rid of the rule (R.GTS), since the clause for substitutional existential quantification of L_1 can be mirrored by a rule in a semantic game, namely by (R.Σ) – as is the case for objectual quantification (because, under certain restrictions – an at most denumerable universe, and an extensional language (see Kripke 1976: 162) – Kripke's and Tarski's accounts of quantifiers are equivalent). We can then restrict the rule for L_1-sentences (R.L_1) to a rule for the basic sentences of L_0:

> ($R.L_0$). If ϕ is a formula of L_0 (i.e. a basic sentence), then the verifier wins G(ϕ) if ϕ is true, and the falsifier wins if ϕ is false.

The truth of complex L_0-sentences can similarly be defined in a game-theoretical style, so that games reach atomic formulas just as in traditional GTS. Hence we can replace ($R.L_0$) by a rule for atomic formulas, namely (R.At). The point here is double: (i) in our substitutional version of GTS, the truth-definition for atomic formulas *need not* be denotational (it can for instance be defined in a syntactical way); (ii) *no objects* are required for the games to be played: only designators (namely: the individual constants of C_1) are needed.

We eventually obtain a new version of the 'mixed' GTS rules for L_{IF}, which can be contrasted with Hintikka's rules:

<table>
<tr><th>Hintikka's rules.
The entire game is played on some given model M of the underlying language…</th><th>'Substitutional GTS' rules
The entire game is played on some given interpretation of L_1, with a (substitution) class of constant symbols C_1.</th></tr>
<tr><td colspan="2">(R.At). If A is a true atomic sentence (or identity), the verifier wins G(A) and the falsifier loses it. If A is a false atomic sentence (or identity), vice versa.</td></tr>
<tr><td colspan="2">(R.Ú). G($S_1 \vee S_2$) begins with the choice by the verifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).</td></tr>
<tr><td colspan="2">(R.Ù). G($S_1 \wedge S_2$) begins with the choice by the falsifier of S_i (i = 1 or 2). The rest of the game is as in G(S_i).</td></tr>
<tr><td>(R.$). G(($\exists x$)$S[x]$) begins with the choice by the verifier of a member of do(M). If the name of the individual is b, the rest of the game is as in G($S[b]$).</td><td>(R.S). G((Σx)$S[x]$) begins with the choice by the verifier of a member a_i of the substitution class C_1. The rest of the game is as in G($S[a_i]$).</td></tr>
<tr><td>(R."). G(($\forall x$)$S[x]$) is likewise, except that the falsifier makes the choice.</td><td>(R.P). G((Πx)$S[x]$) is likewise, except that the falsifier makes the choice.</td></tr>
<tr><td colspan="2">(R.~). G(~S) is like G(S), except that the roles of the two players (as defined by these rules) are interchanged.</td></tr>
</table>

It appears that this new version is quite intuitive as a formulation of Hintikka's rules for semantical-games. There seems to be no principled reason to construct GTS as an extension of standard objectual interpretation rather than as an extension of a substitutional one.[2] Insofar as we don't go beyond denumerable domains, this substitutional account of GTS seems to do the job.

5. DUMMY VARIABLES AND SUBSTITUTION

Unfortunately (again), the so-called 'dummy-variables' would directly threaten this construction. They were discovered by Hodges (1997), and later on Sandu (2000) argued that they should break any presumed equivalence between objectual and substitutional quantifications for IF first-order languages.

Sandu (2001: 24-25) explains that there are two ways of "skolemizing", according to whether a player can have imperfect information about the moves made by the opponent, or about every previous move (made by the opponent or by him- or herself). The first way, which I will call Narrow Skolemization, consists in defining the strategies of a player "on all the possible known moves made earlier in the game by the opponent"; the second way, or Wide Skolemization, "is to define them on all the earlier possible moves, no matter whether they are made by the same player or his opponent".[3]

For ordinary first-order formulas, the two ways are equivalent, but, as Hodges puts it, "certain things which are obviously equivalent for first-order logic split apart when there is imperfect information" (1997: 546).

Let's consider again the sentence (HS-2):

(HS-2). $(\forall x)\,(\forall z)\,(\exists y/\forall z)\,(\exists u/\forall x)\,S[x, y, z, u]$

Then according to Narrow Skolemization we obtain:

(NS-HS2) $\text{(HS-2)} \Leftrightarrow (\exists f)\,(\exists g)\,(\forall x)\,(\forall z)\,S[x, f(x), z, g(z)]$

whereas with Wide Skolemization, we get:

(WS-HS2) $\text{(HS-2)} \Leftrightarrow (\exists f)\,(\exists g)\,(\forall x)\,(\forall z)\,S[x, f(x), z, \boldsymbol{g(z, f(x))}]$[4]

Let's now consider Hodges's formula and its skolemizations:

(Ho). $(\forall x)\ \mathbf{(\$z)}\ (\exists y/\forall x)\ x \neq y$

(NS-Ho) $(Ho) \Leftrightarrow (\exists y)\ (\forall x)\ x \neq y$
[*false* in every model with 2 elements]

(WS-Ho) $(Ho) \Leftrightarrow (\exists f)\ (\exists g)\ (\forall x)\ x \neq f(g(x))$
[*true* in every model with 2 elements]

According to Sandu, "what is going on in substitutional quantification" is that "a formula prefixed with … an existential quantifier is equivalent [to] the sentence which results from the replacement of the relevant variable in the formula with a constant" (2000: 165-166). So, as the existentially quantified variable *z* in Hodges's formula doesn't occur anywhere else in the sentence, it is a "dummy variable" and a substitutional interpretation cannot take it into account. (Ho) formula must thus have the same substitutional interpretation as

(Ho'). $(\forall x)\ (\exists y/\forall x)\ x \neq y$

Consequently, substitutional quantification automatically leads to Narrow Skolemization. But as one can choose the wide option, we have here a case of bifurcation between objectual and substitutional quantification: (Ho) and (Ho') have the same substitutional interpretation, whereas their respective Wide Skolemizations diverge in the following way:

(WS-Ho') $(Ho') \Leftrightarrow (\exists y)\ (\forall x)\ x \neq y$
[*false* in every model with 2 elements]

(WS-Ho) $(Ho) \Leftrightarrow (\exists f)\ (\exists g)\ (\forall x)\ x \neq f(g(x))$
[*true* in every model with 2 elements]

In fact, there is no *a priori* reason why the "dummy variable" should be irrelevant for the substitutional interpretation. It depends on how Kripke's semantics is extended from classical to IF languages, and the mixed solution I am proposing, which is both game-theoretical and substitutional, can deal with such cases.

One might object that the mixed solution is a covert case of objectual interpretation, because it allows referential quantification on choice functions. Against this, it should be noted that such quantifications only occur at a metalinguistic level, when we explicitly speak of the existence of strategies for players in a semantical-game, that is, when we explain the semantics of the language. Moreover, following this argument, Kripke's semantics would be objectual too, since he admits objectual quantifiers in his metalanguage (Kripke 1976: 341).

The only difference between Hintikka's objectual GTS and the substitutional version lies in the definition of the truth-values for the atomic formulas. According to the first account, we need *bona fide* objects, while the second one only requires constant symbols. If, as I think, both versions are equivalent in their upshots (at least for denumerable domains), it would mean that the nature of the semantic attributes of singular terms is irrelevant for game-theoretical semantics. In other words, GTS is *not* essentially linked to objectual quantification. What I will thus later advocate is that a substitutional or syntactical conception of quantifiers better fits the meaning of quantifiers – as it is accounted for by GTS or, more generally, by dynamic semantics (in the broad sense) –, than the received objectual one.

6. SOME PUZZLING FEATURES OF THE OBJECTS IN IF LANGUAGES

Let's add some further remarks about Narrow and Wide Skolemization.

In a model with two or more elements, is Hodges's sentence true or not? Before Hodges's paper, game-theoretical semantics, which was constructed by Hintikka after Narrow Skolemization, would have deemed it false. Now, we can choose Wide Skolemization, so that it is true. So, the truth-value of Hodges's sentence depends on epistemic features. Of course I do not mean it depends on our knowledge of Hodges's paper, but that it depends on the information available to the ideal players of semantical-games. If the verifier can forget his or her own previous moves in the game, then Hodges's sentence is true; if not, it is false.

Another question then immediately arises: What is Hodges's sentence about? In interpreted classical languages, sentences can be conceived of as being about the interpretation structure, that is, about the objects of its domain, their properties and relations; hence sentences are true, false, or undetermined. But in IF languages, it cannot be so clear. Here, it seems that the subject-matter of sentences is not only the model-theoretic structure, but also the information-flow through semantical-games. A "dummy variable" is dummy in the sense that in a classical game, its assignment doesn't provide any contribution to the semantic value of the whole sentence; however a dummy variable does not entail a dummy move but a genuine one, so that for IF languages, because the moves in semantical games are constitutive of the meaning of expressions, a dummy variable does affect the value of the whole sentence. Roughly said, the interpretation structure and its objects do not matter any longer; players can do the job either with objects, or with constant symbols, what matters is their knowledge about the game they play.

Another puzzling fact concerning IF languages is that they cannot have a semantics which would be both compositional and objectual *stricto sensu* (see Sandu and Hintikka 2001). For classical languages, semantic compositionality and objectuality seem to go together in quite a natural way: if semantic values of sentences are to be determined by those (whatever they are) of their component parts and by the global syntactical structure, then it is natural to conceive of values of singular terms as individual objects. Tarskian semantics meets this requirement as it assigns to each formula the set of sequences of objects satisfying it.

But when we go beyond classical languages and build IF languages, we have to choose. Semantic objectuality *can* be maintained, as in Hintikka's game-theoretical semantics, but then we lose semantic compositionality; moreover, no extension of GTS can rescue it for IF languages, whereas it can be restored for classical languages.[5]

An alternative is to retain semantic compositionality, but then we lose objectuality. It is the case with Hodges's *trump semantics* for IF languages, which is compositional, but not objectual in the strict sense: formulas are no longer assigned a set of sequences (or *tuples*) of objects, but a set of *trumps*, that is, a set of sets of sequences. Within *trump semantics*, one just cannot understand or even imagine what kind of "things" are the values of the variables; they are certainly not model-theoretic objects.

However, according to some authors, such as Puntel, semantic sentential compositionality straightforwardly "induces an ontology of *objects* that have properties standing in relations to other objects" (Puntel 2001: 232).[6] (I would like to add: *if* we assume that there must be an ontology, that is, if we are not looking for a substitutional interpretation). For sure, Puntel's claim is about classical languages. But the split between compositionality and objects for IF languages reinforces the idea that IF sentences are not so much about such and such interpretation structures, than about evaluation games.

Hence so-called IF *languages* are not 'languages' in the same sense as classical ones. The claim that they are about games, that is about their own semantics (if we assume that GTS is their primary semantics, and *trump semantics* only secondary), this claim is not metaphorical. Conversely, it means that IF languages *can express* their own semantics – and it is the case.

The game-theoretic truth-condition of a sentence consists in the existence of a winning strategy for the verifier. There is an equivalence between second-order quantifications on choice functions expressing the truth-condition of the first-order sentence, and the sentence itself, such as in the following example:

$$(\forall x)\,(\forall z)\,(\exists y/\forall z)\,(\exists u/\forall x)\,S[x, y, z, u] \Leftrightarrow (\exists f)\,(\exists g)\,(\forall x)\,(\forall z)\,S[x, f(x), z, g(z)]$$

Thanks to the slash notation, an existential assertion about choice functions can be translated into the assertion of a first-order sentence. A metalinguistic assertion about the truth-conditions of an IF-sentence is thus expressed by the sentence itself.[7]

This property indeed indirectly challenges the presumed "objectuality" of GTS for IF languages. Of course, the values of variables are not here so obscure as in *trump semantics*, but they are not as transparent as in classical first-order languages: as one asserts the existence of an individual object, one asserts the existence of a choice function. The so-called 'ontological commitment' of a theory expressed in an IF language would be very confusing.

Finally, whatever semantic interpretation one chooses for IF languages, it seems that one cannot reach any *genuine* objectual account of quantifiers.

7. GTS CANNOT ESCAPE FROM SUBSTITUTION

On the other hand, substitution appears to be better suited for game-theoretical semantics.

First of all, according to both GTS and substitutional semantics, open formulas don't have any semantic values, only sentences do have one, so that compositionality does not hold.

But there is more to come. Let's recall the GTS rules for quantifiers:

> **(R.$)**. G($(\exists x)S[x]$) begins with the choice by the verifier of a member of do(**M**). If the name of the individual is *b*, the rest of the game is as in G($S[b]$).
> **(R.")**. G($(\forall x)S[x]$) is likewise, except that the falsifier makes the choice.

The universally quantified sentence '$\forall x\ S[x]$' is true iff there is a winning strategy for the verifier *for every substitutional instance* '$S[c]$' of the open sub-formula '$S[x]$', where the initial falsifier selects (an object and) its name '*c*'.

Strategies may thus involve infinitely many plays, with infinitely many 'names' or symbol constants. But in continuous universes, one cannot enumerate all the objects. The rules for quantifiers must then allow to add new names or constant symbols for the continuation of the game when the name of a selected object is lacking (it is the case in several formulations of the rules by Hintikka).

But a problem arises here. If we are to evaluate an existential sentence '$(\exists x)Px$' relatively to a structure such that the winning strategy appeals to an object which was not previously named, the verifier must then add a name '*c*' to the language in order to reach the atomic formula '*Pc*'; but this formula does not belong to the language. It has not been previously interpreted. Thus we cannot know whether the verifier is winning or not. Is there any way-out?

In a different context, Lavine (2000) provides a new semantics called 'Geach-Tarski semantics': "'Geach' for the use of the substitutional quantification with the possibility of substituting *new constant symbols* and 'Tarski' for the procedure of interpreting the new constant symbols by assigning them members of the old domain and for defining the truth of the atomic sentences in the expanded languages in standard Tarskian style" (Lavine 2000: 9):

Expansion of a structure

A structure $\mathbf{B} = \langle \text{do}(\mathbf{B}), I_{\mathbf{B}} \rangle$ is an *expansion* of a structure $\mathbf{A}\ \langle \text{do}(\mathbf{A}), I_{\mathbf{A}} \rangle$ if:

(i) **A** and **B** have the same domain, i.e. $\text{do}(\mathbf{A}) = \text{do}(\mathbf{B})$;

(ii) The signature $S_{\mathbf{A}}$ of the language of **A** is a subset of that of the language of **B**, $S_{\mathbf{B}}$;

(iii) The interpretation function $I_{\mathbf{B}}$ agrees with $I_{\mathbf{A}}$, on the signature $S_{\mathbf{A}}$.

Recursion clause for existential quantification

If ϕ is a formula for the language L of the structure **A**, and *c* a constant symbol which is not in the signature $S_{\mathbf{A}}$ of L, then:

(i) $(\exists x)\phi$ is true over the structure **A** iff there is an expansion **B** of **A** to the language of signature $L \cup \{c\}$ such that $[c/x]\phi$ is true over the structure **B**.

(ii) $(\forall x)\phi$ is true over the structure **A** iff for every expansion **B** of **A** to the language of signature $L \cup \{c\}$, $[c/x]\phi$ is true over **B**.

Lavine explains: "From a mathematical perspective, the proposed substitutional semantics for quantification is a trivial variant of the earlier referential one … The procedure yields the same truth values as the more standard one by essentially the same method" (*op.cit.*: 8-9).

Between the two semantic theories, there is a shift in the referential device (this is in fact Lavine's purpose): variables, which are thought of as referring in Tarski's semantics because of the assignments, do not refer any more in Lavine's, but constant symbols take over.

The interesting fact here is that, although the theories are equivalent, GTS can be based on Lavine's semantics, but not on Tarski's. To put it in other words: GTS can be based on a substitutional interpretation of quantifiers, but not on the standard objectual one. The existential rule (R.E) should be changed into a new one, (R.E*), which would explicitly mention the structure on which the game is played:

> **(R.E*)**. G(**M**, (∃*x*)*S*[*x*]) begins with the choice by the verifier of a member of do(**M**). If the name of the individual is *b*, and if it is already in the language, the rest of the game is as in G(**M**, *S*[*b*]); if the name is not yet in the language, the rest of the game is as in G(**M'**, *S*[*b*]), where **M'** is an expansion of the structure **M**.

It might be objected that Lavine's semantics is fundamentally an objectual one, because a domain of objects is still needed. In fact, it is possible to get rid of these objects, so that we yield a full-fledged substitutional semantics. Lavine says:

> "There are variant semantics that differ from what we have been considering only in the way the atomic sentence are handled; we keep the Geach half of Geach-Tarski semantics and change the Tarski part. Instead of interpreting relation symbols by relations and constant symbols by objects, one could give syntactic tests for the truth of atomic sentences." (*op.cit.*: 17).

Once more, the very nature of the semantic correlates of constant symbols is irrelevant for GTS – they can be objects, or linguistic items; what matters here is the account of quantified sentences, and because semantical-games end with atomic sentences, this account is required to be a substitutional one.[8]

8. DYNAMIC SEMANTICS AND SUBSTITUTION

Let's now turn to natural language semantics. Here, one can find more fundamental reasons to choose some substitutional account of quantifiers.

To put it in a nutshell: variables standing for indefinite descriptions need not refer, above all when nothing fits the description, in order to do their job in anaphora resolution. And the question carries over to other categories of singular expressions, such as empty proper names.

Without going into technical details, let's briefly survey how two competing semantic theories, namely (standard objectual) GTS and Kamp's Discourse Representation Theory (DRT), would account for a series of examples of cross-sentential anaphoric relations:

(1.a) John walks in the park. He whistles.
(1.b) *John walks in the park. She whistles.
(1.c) *He whistles. John walks in the park.

(1.a), (1.b) and (1.c) are classical instances of what every 'dynamic' semantic theory must account for. Here, both GTS and DRT can of course do the job. According to GTS, in (1.a), an object of the domain (John) is selected by the verifier and put into a *choice set* while playing the game associated with the first sentence, so that it can be used later in the game, when the second sentence is played; there is thus a winning strategy for the verifier in the game associated with (1.a), but not for (1.b) and (1.c). According to DRT, it is not an object, but a *reference marker* which is introduced into the universe of a discourse *representation* structure.

New questions arise concerning the following examples:

(2.a) Cinderella smiles. She is beautiful.
(2.b) *Cinderella smiles. He is beautiful.
(2.c) *She is beautiful. Cinderella smiles.

Here, the ontological neutrality of DRT representatives is a great advantage: DRT can indeed succeed for this series as with the preceding one. But not standard GTS, which can only succeed if there is an object called Cinderella which can be picked up in the domain to be put into the choice set. It means that if there is no object designated by 'Cinderella', as we adults usually believe, (2.a) cannot be processed.[9] However, what matters in anaphora resolution is not whether there is an individual called Cinderella or not: the important thing is that there is an expression, namely a (pseudo-) proper name, which provides an antecedent for the anaphoric pronoun 'she'.

Similarly, DRT can account for anaphoric binding in (3.a) and (3.b):

(3.a) A farmer owns a donkey. He beats it.

(3.b) A goddess has a daughter. She smiles at her.

whereas (on some non-mythical domain) GTS will fail on the second one. Moreover, the case of indefinite descriptions reveals a really counter-intuitive character of objectual GTS: as the expression ‘a farmer’ is processed in the game, some (perfectly definite) individual has to be picked up from the domain, although no knowledge of the individual is necessary to understand the expression; on the other hand, a substitutional account would only require an individual constant (i.e. nothing more than a discourse referent).

Concerning natural language conditionals, GTS cannot discriminate between (4.a), (4.b) and (4.c), which are trivially resolved in the same manner as successful anaphoric relationships:

(4.a) If a goddess comes into the room, she will smile.
(4.b) *If a goddess comes into the room, he will smile.
(4.c) If a Smurf drives a car, it will be a blue one.

However, (4.a) and (4.c) don’t have the same meaning, and there’s a real problem with (4.b).[10]

It appears that if we want semantic theories to fully account for anaphora, they must be allowed to directly *change* the context. Standard objectual GTS is thus seriously handicapped with its *static* model-theoretic domain. In order to achieve a completely dynamic account of anaphora, GTS should be liberalized so that ideal players can *stipulate* new objects, which would lead us to revise our conception of what are model-theoretic objects.

Another solution would be provided by a substitutional version of GTS. If we start with a finite substitution class of constant symbols, the ideal verifier can, when playing the game associated with the sentence “A man is walking”, resort to a constant symbol not yet used so that the resulting atomic sentence is considered true. And the same process would take place with Cinderella, goddesses and Smurfs. GTS would then yield the same results as DRT (at least with the examples just mentioned), and this *without* any representational level, and *without* any model-theoretic structure, but only with games.

9. CONCLUSION

I plainly agree with Peregrin’s claim that we should fundamentally conceive of dynamic semantics as a matter of inference-patterns, that is as a matter of *syntax* – if we understand it in the broad sense of a handling of symbols rather than in the sense of an axiomatic theory.

Moreover, if traditional 'denotations' can be reduced to "clamp[s] holding certain inferentially related expressions together" (Peregrin 2000), it means that we have to think of the language–world relations as an extra-semantic matter.

As a result, reverting to the substitutional interpretation of quantifiers does not imply keeping out of the development of dynamic semantics. On the contrary, I have argued that some substitutional account of GTS may be worked out which can deal with both IF sentences and anaphora resolution; furthermore, substitution appears to be already covertly present within standard GTS.

The underlying idea of substitution is not that of a strictly syntactic (and static) approach.[11] It is rather the following: it is not the objects we are speaking about, but the linguistic devices we use in order to designate them, which are semantically relevant.

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NOTES

[1] According to Kripke's terminology, L_0-sentences are generally called 'atomic' (see Kripke 1976: 329). But such a labeling would be misleading for our purpose, since L_0-sentences involve every variable-free sentence of the predicate calculus.

[2] Even though it seems to contradict Hintikka's own conception of GTS as an implementation of Wittgenstein's language-games. Nevertheless, some language-games are better accounted for when objectual interpretation is given up (cf. Section 8).

[3] Pietarinen and Sandu (2000: 150) suggest that games with imperfect recall, corresponding to Wide Skolemization, be accounted for game-theoretically by "viewing players as teams of players".

[4] Another way to stress the distinction is to systematically express dependences and independences of quantifiers on existentially *and universally* quantified variables, as Hodges suggests (1997: 549). We then have only Wide Skolemization, and we can rescue an equivalent of Narrow Skolemization of HS-2 with Wide Skolemization of: $(\forall x)\ (\forall z)\ (\exists y/\{z\})\ (\exists u/\{x, y\})\ S[x, y, z, u]$).

[5] Assuming the Axiom of Choice (AC), GTS for classical languages (i.e. with perfect information) can be extended to a (compositional) *strategy-interpretation* (Hodges 1997); without AC, it can be extended to a (compositional) *quasistrategy-interpretation*, which is "nothing more than the Tarski-interpretation in the game-theoretical jargon" (Sandu and Hintikka, 2001).

[6] With non-compositionality, one can provide another type of ontology. In his paper, Puntel uses a non-compositional semantics and reaches an ontology of elementary *facts*.

[7] Furthermore, under certain conditions, the truth-predicate of an IF language can be defined in the very same language.

[8] Another way to solve the issue of missing names is to play semantical-games relatively to a structure *and* an assignment. Sandu (1997: 156) provides such a formulation of the GTS rules. Semantical-games can thus end up with atomic *open* formulas. As a consequence, it seems that we lose a strong intuitive feature of semantical-games, namely that they supply examples of natural language-games as they are thought of by Hintikka: "[Semantical-games] supply examples of language-games of precisely the sort we expect to find but do not in Wittgenstein. They illustrate the function of language-games as giving a meaning also to nonmodal words" (Hintikka 1976: 17). This is so because in our natural language-games, we are only faced with sentences, never with open formulas.

[9] Non-classical GTS can account for such cases, with no resort to choice sets (see Janasik and Sandu's paper in this volume).

[10] According to the GTS account, the verifier and the opponent play the antecedent with their roles reversed. If the verifier wins, he or she has won the whole game; if the opponent wins, the game goes on – with their normal roles – and the player who wins the second subgame wins the whole game.

[11] Substitution can indeed be viewed as a definite assignment in a program, i.e. as a genuine dynamic notion. An example is provided by Venema (1995) who introduces specific "substitution operators" in a modal interpretation of first-order logic.

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